EXTENDED RESULTS ON OPTIMAL INVESTMENT STRATEGIES IN SHRIMP FISHING

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ABSTRACT

A method of obtaining optimal investment strategies for shrimp fishermen is developed and illustrated in this report. The method is designed to enable a shrimp fisherman having a given amount of physical and money capital to obtain guidelines for investment and financial decision-making.

The basis for the method is a deterministic optimal control model of a shrimp fishing firm. This model is derived and explained. Then the parameter values and initial state values which characterize the environment in which the shrimp fishing firm is assumed to be operating are defined and discussed. Once the model has been completely delineated by specifying the parameter values and initial state values, the determination of an optimal investment strategy becomes a mathematical programming problem which can be solved by any of the commercial mixed integer programming computer programs.

Three numerical examples are presented and discussed.

The method may be used to obtain guidelines for the shrimp fishing industry in general or an individual firm. The computer costs to an individual seeking guidelines for his specific fishing environment and initial asset position should generally be less than $25 per year.

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1. Introduction

In April 1970, Thompson, Callen and Wolken published the first of three previous Texas A&M University Sea Grant reports concerning optimal investment strategies in the shrimp fishing industry. That first bulletin [1] contained a deterministic optimal control model of a shrimp fishing firm in addition to much background information on the industry and justification for the model specification. The second publication [2] extended the first model by incorporating unknown, but random, future shrimp prices and catches and a constraint that required solvency to be maintained with a high probability based on the probability distributions of the random prices and catches.

In the third report [3] the original deterministic model was extended to require the purchase of integer numbers of vessels. Fractions could be purchased in the original application [1], but industry representatives suggested that a more realistic specification would require the purchase of integer numbers of vessels. This extension is significant in cases in which holding companies cannot be readily formed to overcome capital indivisibilities. Integer requirements clearly restrict the growth of the firm's physical capital and, consequently, its net worth over a finite planning horizon. If holding companies could be utilized without additional cost, vessel owners could clearly experience a faster rate of net worth accumulation. However, because capital indivisibilities have not generally been overcome in the shrimp fishing industry, the integer restriction is necessary for the model to be reflective of industry conditions.

This paper considers essentially the same model as described in the third report [3] but extends the numerical examples to include alternative sizes of boats. In addition, some relatively minor mistakes in the computer routine used to generate the numerical example in [3] have been corrected, and the example is presented here in corrected form.

In each example the optimal strategy is compared with the conservative strategy of purchasing no additional fishing capacity and retaining all cash flows net of debt repayment as savings. These comparisons illustrate the importance of following an optimal strategy.

2. Dynamic Model for a Shrimp Fishing Firm

The definition of each parameter and variable used to describe the model for a shrimp fishing firm is given as it is introduced and is also summarized in Appendix A.

In the model, the objective of the fisherman is to maximize the amount of savings held in the last year of the decision-making period, z_T, less the amount of indebtedness outstanding at that time, y_T, plus the value of the boats owned in the last year, $\sum_{t=0}^{T} \psi_t \tau_t v_t$, with an allowance being made for technological depreciation, ψ_t, and inflation in the purchase price, τ_t. There are three sets of difference equations and also three sets of inequality restrictions limiting the size of this objective. Furthermore, the number of boats purchased in year t, v_t, is restricted to be a non-negative integer.

Indebtedness, y_t, savings, z_t, and boats owned, x_t, are the state (stock) variables in the model for year t; boat purchases, v_t, and borrowings, w_t, are the control (flow) variables for year t. Initial values of the state variables--number of boats, x_0, indebtedness, y_0, and savings, z_0--are taken as given; the values of the other state variables and the control variables are determined from the optimal investment strategy.

In the model, the shrimp fisherman is allowed to purchase boats but is not allowed to sell boats. Since some time is generally necessary between the time when the decision is made to buy a boat and the boat is operational, the number of boats operated in year t was specified to be the number owned at the end of year t-1; and, accordingly, boat purchases in the last year of the planning period were specified to be zero, $v_T = 0$. Thus, the change in the number of boats owned is described as follows:

$$
\begin{aligned}
& x_t - x_{t-1} = v_t \,, \quad t = 1, 2, \ldots, T-1 \quad , \\
& X_T - X_{T-1} = 0 \quad .
\end{aligned}
\tag{2.1}
$$

In the model, if the fisherman chooses to borrow in year t, he cannot borrow more than a fraction of the value of the boat investment in that year. That is, the fishing firm can only borrow money for the purchase of new boats, and in every case the fisherman must have enough savings in the bank to cover the difference between the maximum loan value and the investment in boats. Letting κ denote the maximum fraction of the boat investment that can be borrowed, the upper limit for borrowings in year t is $\kappa\tau_t v_t$. Thus, the inequality restrictions on w_t are

$$0 \leq w_t \leq \kappa\tau_t v_t \ , \quad t = 1, 2, \ldots, T \quad . \tag{2.2}$$

These restrictions mean that in any year t borrowings, which must clearly be non-negative, may occur only if new boats are purchased, and then they cannot exceed the fraction κ of the investment $\tau_t v_t$. Of course, since v_T is specified to be zero, these restrictions imply that w_T must also be zero.

In each year t, the fisherman is required to repay a fraction, β, of the indebtedness owed at the end of the previous year. Thus, the change in indebtedness is as follows:

$$\begin{aligned} y_t - y_{t-1} &= w_t - \beta y_{t-1} \ , \quad t = 1, 2, \ldots, T-1 \ , \\ y_T - y_{T-1} &= -\beta y_{t-1} \ . \end{aligned} \tag{2.3}$$

To describe the fishing firm's cash flow, it is helpful to have the following symbols: γ_t is the net price per pound of heads-off shrimp received by the owner in year t after the lay is paid; λ is the expected catch per boat in pounds of heads-off shrimp; η_t is the sundry expense associated with the fishing operation in year t; ζ is the interest rate paid on debt; ξ is the interest rate earned in savings; σ is the income tax rate; θ_t is the cost of operating a fishing boat in year t; and $g_t(v_i)$ is the depreciation allowed in year t on the boats purchased in year i. Then the difference equations describing the firm's cash flow are:

$$z_t - z_{t-1} = w_t - \beta y_{t-1} - \eta_t - \tau_t v_t + (\gamma_t \lambda - \theta_t) x_{t-1} - \zeta y_{t-1}$$

$$+ \xi z_{t-1} - \sigma[(\gamma_t \lambda - \theta_t) x_{t-1} - \eta_t - \zeta y_{t-1} + \xi z_{t-1}$$

$$- \sum_{i=0}^{t-1} g_t(v_i)] \quad , \qquad t = 1, 2, \ldots, T-1 \quad , \tag{2.4}$$

$$z_T - z_{T-1} = -\beta y_{T-1} + (\gamma_T \lambda - \theta_T) x_{T-1} - \eta_T - \zeta y_{T-1} + \xi z_{T-1}$$

$$- \sigma[(\gamma_T \lambda - \theta_T) x_{T-1} - \eta_T - \zeta y_{T-1} + \xi z_{T-1} - \sum_{i=0}^{T-1} g_T(v_i)] \quad .$$

In every year except the last one, the cash flow or change in savings is equal to the change in indebtedness less the boat investment plus the earnings retained after taxes. Before tax earnings equal net revenues to the boat owner and interest earnings on savings less interest payments on debt. In calculations in this paper discounted net profits after taxes will be regarded as the retained earnings after taxes. Such a definition implies that no personal allowances are used from the earnings in case the ownership is non-corporate and that no dividends are declared if ownership is corporate. If a boat is owner-operated, of course, the captain's share of the lay also goes to the owner and is an additional element of profit that our definition overlooks.

Initially, the fishing firm is regarded as having a given amount of fishing capacity, $x_0 > 0$, with possibly some indebtedness, $y_0 \geq 0$. It may or may not have any savings at the beginning of the period, $z_0 \geq 0$.

The parameters in the model, which are denoted by Greek letters, are all positive with σ, ζ, ξ, β, and κ being less than unity. It is also assumed that $\zeta > \xi$.

2.1 Mathematical Statement of the Decision-Making Model

The model described above can be formally stated as the following discrete-time control problem: Given x_0, y_0, z_0, $v_0 = x_0$, and $v_T = 0$

$$\text{maximize } z_T - y_T + \sum_{i=0}^{T} \psi_i \tau_i v_i \tag{I.1}$$

subject to the difference equations

$$\begin{aligned} x_t - x_{t-1} &= v_t \quad , \\ x_T - x_{T-1} &= 0 \quad , \end{aligned} \tag{I.2}$$

$$\begin{aligned} y_t - y_{t-1} &= w_t - \beta y_{t-1} \quad , \\ y_T - y_{T-1} &= -\beta y_{T-1} \quad , \end{aligned} \tag{I.3}$$

$$z_t - z_{t-1} = w_t - \beta y_{t-1} - \tau_t v_t + (\gamma_t \lambda - \theta_t) x_{t-1} - \eta_t$$

$$- \zeta y_{t-1} + \xi z_{t-1} - \sigma[(\gamma_t \lambda - \theta_t) x_{t-1} - \eta_t - \zeta y_{t-1} + \xi z_{t-1}$$

$$- \sum_{i=0}^{t-1} g_t(v_i)] \quad , \qquad \text{(I.4)}$$

$$z_T - z_{T-1} = -\beta y_{T-1} + (\gamma_T \lambda - \theta_t) x_{T-1} - \zeta y_{T-1} + \xi z_{T-1} - \eta_T$$

$$- \sigma[(\gamma_T \lambda - \theta_T) x_{T-1} - \eta_T - \sum_{i=0}^{T-1} g_T(v_i) - \zeta y_{T-1} + \xi z_{T-1}] \quad ,$$

and the restrictions

$$w_t \geq 0 \quad , \quad t = 1, 2, \ldots, T-1 \quad , \qquad \text{(I.5)}$$

$$w_t \leq \kappa \tau_t v_t \quad j \quad , \quad t = 1, 2, \ldots, T-1 \quad , \qquad \text{(I.6)}$$

$$z_t \geq 0 \quad , \quad t = 1, 2, \ldots, T \quad , \qquad \text{(I.7)}$$

$$v_t = \text{a non-negative integer} \quad , \quad t = 1, 2, \ldots, T-1 \quad . \qquad \text{(I.8)}$$

Letting $g_t(v_i) = \tau_i v_i/(T+1)$ and solving the difference equations in (I.2), (I.3), and (I.4) for their respective "closed-form" solutions, the state variables can be stated in terms of their initial values and the unknown control variables:

$$x_t = x_0 + \sum_{i=1}^{t} v_i \quad , \tag{2.5}$$

$$y_t = y_0(1-\beta)^t + \sum_{i=1}^{t} w_i(1-\beta)^{t-i} \quad , \tag{2.6}$$

$$\begin{aligned} z_t = z_1 Q_{t1} + \sum_{i=2}^{t} [w_i - \tau_i v_i + \Delta_i x_{i-1} + \pi y_{i-1} + \sigma \sum_{j=0}^{i-1} \tau_j v_j/(T+1) \\ + (\sigma-1)\eta_t]Q_{ti} \quad , \end{aligned} \tag{2.7}$$

where for i = 1, 2, ..., t and t = 1, 2, ..., T

$$\Delta_i = (\gamma_i \lambda)(1-\sigma) - (1-\sigma)\theta_i \quad ,$$

$$\pi = \zeta(\sigma-1) - \beta \quad ,$$

$$Q_{ti} = (1+\Gamma)^{t-i} \quad ,$$

$$\Gamma = \xi(1-\sigma) \quad ,$$

and

$$v_T = 0$$

$$w_T = 0 \quad .$$

Substituting the closed-form solutions for x_t and y_t from (2.5) and (2.6) into (2.7), we obtain the following solution for z_t in terms of the initial state values, the unknown controls, and the parameters:

$$z_t = C_t + \sum_{i=1}^{t} w_i P_{ti} + \sum_{i=1}^{t} v_i D_{ti} \quad , \quad t = 1, 2, \ldots, T-1 \quad , \tag{2.8}$$

where

$$C_t = \sum_{i=1}^{t} Q_{ti}\{[\Delta_i + \sigma\tau_0/(T+1)]x_0 + (\sigma-1)\eta_i\}$$

$$+ \pi y_0 \sum_{i=1}^{t} Q_{ti}\, \chi^{i-1} + (1+\Gamma)z_0\, Q_{t1} \quad , \quad t = 1, 2, \ldots, T-1 \quad ,$$

$$\chi = 1 - \beta \quad ,$$

$$P_{tt} = Q_{tt} \quad , \quad t = 1, 2, \ldots, T-1 \quad ,$$

$$P_{ti} = Q_{ti} + \pi \sum_{j=i+1}^{t} Q_{tj} R_{j-1,i} \quad ,$$

$$i = 1, 2, \ldots, t-1 \text{ and } t = 2, \ldots, T-1 \quad ,$$

$$D_{tt} = -\tau_t Q_{tt} \quad , \quad t = 1, 2, \ldots, T-1 \quad ,$$

$$D_{ti} = \sum_{j=i+1}^{t} \Delta_j Q_{tj} + [\sigma\tau_i/(T+1)] \sum_{j=i+1}^{t} Q_{tj} - \tau_i Q_{ti} \quad ,$$

$$i = 1, 2, \ldots, t-1 \text{ and } t = 2, 3, \ldots, T-1 \quad ,$$

$$R_{ti} = (1-\beta)^{t-i} \quad , \quad i = 1, 2, \ldots, t \text{ and } t = 1, 2, \ldots, T-1 \quad ;$$

and

$$z_T = \sum_{i=1}^{T-1} w_i P_{Ti} + \sum_{i=1}^{T-1} v_i D_{Ti} + C_T$$

where

$$C_T = (1+\Gamma)C_{T-1} + \tau_0 x_0 \sigma/(T+1) + (\sigma-1)\eta_T + \Delta_T x_0 + \pi y_0 \chi^{T-1} \quad ,$$

$$P_{Ti} = \pi R_{T-1,i} + (1+\Gamma)P_{T-1,i}, \quad i = 1, 2, \ldots, T-1 \quad ,$$

$$D_{Ti} = \Delta_T + (1+\Gamma)D_{T-1,i} + \sigma\tau_i/(T+1), \quad i = 1, 2, \ldots, T-1 \quad ,$$

$$R_{Ti} = (1-\beta)^{T-i}, \quad i = 1, 2, \ldots, T \quad .$$

2.2 The Mathematical Programming Problem

Substituting the above solutions for the state variables--x_t, y_t, z_t--into the objective function and the inequality restrictions of the control problem, the state variables and the difference equations describing them are removed from the problem. The resulting problem is the following mathematical programming problem:

$$\text{Maximize} \quad a + \sum_{t=1}^{T-1} B_t v_t + \sum_{t=1}^{T-1} A_t w_t \tag{II.1}$$

subject to the inequality restrictions

$$w_t \geq 0 \qquad t = 1, 2, \ldots, T-1 \quad , \tag{II.2}$$

$$\kappa\tau_t v_t - w_t \geq 0 \quad , \qquad t = 1, 2, \ldots, T-1 \quad , \tag{II.3}$$

$$\sum_{i=1}^{t} P_{ti} w_i + \sum_{i=1}^{t} D_{ti} v_i \geq -C_t \quad , \qquad t = 1, 2, \ldots, T \quad , \tag{II.4}$$

$$v_t = \text{a non-negative integer}, \ t = 1, 2, \ldots, T-1 \quad , \tag{II.5}$$

where

$$A_t = P_{Tt} - R_{T-1,t}\,(1-\beta), \quad t = 1, 2, \ldots, T-1 \quad ,$$

$$B_t = D_{Tt} + \psi_t \tau_t, \quad t = 1, 2, \ldots, T-1 \quad ,$$

$$a = C_T + \psi_0 \tau_0 x_0 - y_0\,\chi^T \quad ,$$

$$w_T = 0 \quad \text{and} \quad v_T = 0 \quad .$$

Letting

$$h_t \equiv h_t(w_1, \ldots, w_{t-1}, v_1, \ldots, v_{t-1}) \equiv C_t + \sum_{i=1}^{t-1} P_{ti} w_i$$

$$+ \sum_{i=1}^{t-1} D_{ti} v_i, \quad t = 1, 2, \ldots, T \quad ,$$

inequality (II.4) may be expressed as follows in terms of the non-negative function h_t:

$$w_t - \tau_t v_t + h_t \geq 0 \ , \quad t = 1, 2, \ldots, T \quad .$$

3. Using the Model to Obtain an Investment Strategy

In this section three numerical examples are presented in order to illustrate how a shrimp fisherman having a given amount of physical and money capital can use the model to obtain guidelines for investment and financial decision-making.

First, the parameter values and initial state values for each of the three examples are specified and discussed. These values characterize the environment in which the shrimp fishing firm is assumed to be operating. They also transform the general decision-making model described in Section 2 into one explicitly pertaining to that environment and completely delineate the mathematical programming problem (II.1-II.5) which, when solved, will yield the optimal investment strategy for a shrimp fishing firm operating in that environment.

Once the mathematical programming problem (II.1-II.5) has been completely delineated by specifying the parameter values and initial state values, it is a problem which can be solved by any of the commercial mixed integer programming computer programs. Such a program was used to solve the three mathematical programming problems corresponding to the three sets of parameter values and initial state values. The solutions (optimal investment strategies) obtained are discussed following the description of the parameter values and initial state values used in the examples.

Table 1. Parameter Values and Initial State Values for Example 1, 2, and 3

Parameter	Interpretation	Value		
		Example 1	Example 2	Example 3
T	Length of planning period	10 years	10 years	10 years
	Length of boat	61 feet	73 feet	90 feet
x_0	Initial number of boats	1	1	1
y_0	Initial indebtedness	\$51,935	\$75,000	\$93,750
z_0	Initial savings	\$ 5,000	\$ 5,000	\$ 5,000
κ	Maximum fraction of the boat's price that can be borrowed	.75	.75	.75
β	Debt repayment rate	.10	.10	.10
ζ	Interest rate on debt	.095	.095	.095
ξ	Interest rate on savings	.055	.055	.055
γ_t	Net price per pound of heads-off shrimp in year t in dollars	$.72416(1.03)^t \cdot \exp(.0176t)$	$.72416(1.03)^t \cdot \exp(.0176t)$	$.72416(1.03)^t \cdot \exp(.0176t)$
λ	Expected catch per boat in pounds of heads-off shrimp	52,787	57,560	75,000
τ_t	Purchase price of a boat in year t	$\$69,246(1.03)^t$	$\$100,000(1.03)^t$	$\$125,000(1.03)^t$
ψ_t	Technological depreciation factor: fractional value of a boat purchased in year t at the end of the planning period	$(1/1.044)^{11-t}$	$(1/1.044)^{11-t}$	$(1/1.044)^{11-t}$
θ_t	Operating cost per boat in year t	$\$26,792(1.03)^t$	$\$30,000(1.03)^t$	$\$37,000(1.03)^t$
η_t	Sundry expense in year t	$\$\ 1,200(1.03)^t$	$\$\ 1,200(1.03)^t$	$\$\ 1,200(1.03)^t$
σ	Income tax rate	.25	.25	.25

3.1 Parameter Values and Initial State Values for the Examples

The parameter values and initial state values used in the three examples reflect existing conditions in 1969 and were derived from information supplied by cooperating shrimp fishing firms operating in the Gulf of Mexico. These values are given in Table 1.

In the examples the lengths of the planning period are the same, $T = 10$, and the initial state values are similar. The firm is initially operating one steel hull trawler of specified length; i.e., $x_0 = 1$. The initial indebtedness is $y_0 = (1-\kappa)\tau_0$, and the initial savings is $z_0 = \$5,000$.

The loan contract in each example states that the maximum fraction of the boat investment which can be borrowed is $\kappa = .75$ and requires the indebtedness to be repaid at a rate of 10% yearly starting at the end of the first year with interest (including mortgage insurance) at 9 1/2 percent annually; $\beta = .10$ and $\zeta = .095$. The interest rate on savings is specified to be 5 1/2 percent annually, the present maximum rate on savings deposits; $\xi = .055$.

Since it is quite common for owners of vessels like these to obtain 65 percent of the gross revenues with the captain and first mate (who pay for all of the groceries) receiving the other 35 percent, the net price per pound of heads-off shrimp landed is specified to be 65 percent of the price in year t. The exvessel price for shrimp in year t, ε_t, was determined by the equation developed in Thompson et al. [2, p. 10]:

$$\ln(100\ \varepsilon_t) = 4.4687 + 0.0176t \quad .$$

The above equation gives estimates of the exvessel average price of shrimp with landings at the mean value of the period 1958 through 1967 and projected 1.5 percent rate of growth in real per capita income. The 1.5 percent rate of growth in real per capita income reflects the slow rate of growth of the late 1950's. This rate of growth appears reasonable as opposed to a faster rate of growth observed in the middle 1960's. To convert to money terms, the projected exvessel prices from this equation are multiplied by the value of the consumer price index (with base 1957/59 = 100) for 1969, 1.277, and by a price inflating factor of 3.0 percent in each year thereafter. Thus, the net price per pound of heads-off shrimp in year t is

$$\gamma_t = (1.03)^t \,(1.277)(.65)\,[\exp(4.4687 + 0.0176t)]/100 \quad ,$$

or

$$\gamma_t = .72416\,(1.03)^t \exp(.0176t) \quad .$$

The prices thus determined are listed in Table 2.

The expected annual landings per vessel, λ, used in each example was the average of the landings per vessel of that size obtained by the cooperating firms in the period 1958 through 1969. There was, of course, a steady rate of technological improvement in that period so that these averages are likely to be conservative estimates of the vessels' annual catch potentials.

In determining the net worth of the firm at the end of year i the value of a boat purchased in year t is assumed to be

$$(1/1.044)^{i+1-t}\ \tau_t$$

and is based on the argument in Thompson et al. [1, p. 29]. The factor $(1/1.044)^{i+1-t}$ represents the technological depreciation. Thus, in determining the net worth at the end of the planning period the technological depreciation factor for a boat purchased in year t is

$$\psi_t = (1/1.044)^{T+1-t} = (1/1.044)^{11-t} \quad ,$$

and is given in Table 2.

Representatives of the firms interviewed indicated that their costs have increased by 3 percent per year in recent years. Thus, an inflation factor, $(1.03)^t$, is included in the operating cost, θ_t, for year t and also in the purchase price, τ_t, of a new vessel in year t.

In shrimp fishing, the captain and first mate of the vessel are commonly paid on a "lay" basis wherein they receive an agreed upon percentage of the revenue earned by the vessel. The remaining crew members, who are called headers, are typically paid on a per box basis. An allowance for their wages is included in the value of the yearly operating cost per vessel.

As in every business, there are sundry expenses for a number of factors related to the firm. Some of these costs, it might be argued, are not absolutely necessary for the operation of the business; but, for the sake of convenience (or acceptance), they are commonly incurred. Such costs are difficult to estimate. Thus, in this study, a yearly allowance is specified for sundry expenses; $\eta_t = \$1,200(1.03)^t$.

Income for tax purposes is the sum of the revenue received by the owner after the "lay" less operating costs, interest costs, and depreciation. The income tax rate, which is denoted by σ, was taken to be 25 percent of this

Table 2. Net Prices Per Pound of Heads-Off Shrimp and the Vessel's Technological Depreciation Factor

t	γ_t	ψ_t
0		.623
1	\$ 0.759	.650
2	\$ 0.796	.679
3	\$ 0.834	.709
4	\$ 0.875	.740
5	\$ 0.917	.772
6	\$ 0.961	.806
7	\$ 1.008	.842
8	\$ 1.056	.879
9	\$ 1.107	.917
10	\$ 1.161	.958

figure. This rate was paid in the late 1960's by a number of the small fishing firms studied.

3.2 The Optimal Investment Strategies Obtained

The solutions to the three mathematical programming problems corresponding to the parameter values and initial state values specified for Example 1, 2, and 3 are given in Table 3, 5, and 7, respectively. For comparative purposes, the corresponding information for the sub-optimal conservative investment strategy of purchasing no additional fishing capacity and retaining all cash flows net of debt repayment as savings is given in Table 4, 6, and 8.

In Example 1 the optimal strategy calls for 1, 1, 1, 2, and 3 boats to be purchased in years 4, 6, 7, 8, and 9, respectively,and for borrowings of \$57,028, \$53,082, \$60,979, \$131,578, and \$203,287 to be made in years 4, 6, 7, 8, and 9, respectively. This strategy results in a net worth of \$332,038 at the end of the last year which is \$203,430 more than the corresponding net worth using the conservative strategy, \$128,608.

The optimal strategies in Example 2 and 3 also significantly bettered their corresponding conservative strategies. The improvement in final net worth was \$27,786 and \$87,295 in Example 2 and 3 respectively.

4. Remarks

Our objective in this paper has been to illustrate a method of obtaining optimal investment strategies for shrimp fishermen. The objectives of our Sea Grant Research Project have included (1) the development of models of optimal investment decisions in shrimp fishing; (2) the refining of those models to be reflective of industry conditions and practices and be practicable as a management tool; and (3) to disseminate the information for use by fishermen. The first objective was previously accomplished. This paper has been concerned with objectives 2 and 3.

Guidelines may be obtained for the industry in general by using hypothetical initial conditions and parameter values. However, exact prescriptions for any given firm can and should be obtained using that firm's particular initial asset position and it's own parameter values. Computer costs for an individual prescription based upon the general dynamic model of a shrimp fishing firm described in section 2 should generally be less than $25 per year given that the firm can ascertain its own parameter values and initial asset position. The total computer cost for all three examples described herein was only $33.62.

Some additional refinements in the dynamic model of a shrimp fishing firm are possible. For example, the possibility of selling old boats or trading them in could be included. The possibility of simultaneously operating more than one size of vessel could also be included.

The dynamic stochastic model described in [2] differed from the model presented here in that prices and catches did not have to be known in advance, and the number of boats purchased in a year was not restricted to be an integer. The dynamic stochastic model learns the prices and catches in each harvesting period, just as the shrimp fisherman does. Thus, random or actual sequences of prices and catches could be utilized to obtain optimal decision rules. The integer refinement along with the other refinements mentioned up to this point could also be implemented with the dynamic stochastic model.

Table 3. Optimal Investment Strategy in Example 1

Year	States			Controls		Objective
	Boats Owned	Indebtedness	Savings	Boats Purchased	Borrowings	Net Worth
	x_t	y_t	z_t	v_t	w_t	
	(number)	(dollars)	(dollars)	(number)	(dollars)	(dollars)
0	1	51,935	5,000	1	51,935	19,393
1	1	46,742	6,319	0	0	23,110
2	1	42,067	9,386	0	0	28,173
3	1	37,861	14,233	0	0	34,662
4	2	91,103	0	1	57,028	39,383
5	2	81,992	12,712	0	0	55,706
6	3	126,876	0	1	53,082	72,028
7	4	175,167	3,730	1	60,979	100,671
8	6	289,228	1,509	2	131,578	140,963
9	9	463,593	0	3	203,287	206,634
10	0	417,233	107,276	0	0	332,038

Table 4. Conservative Investment Strategy in Example 1

Year	States			Controls		Objective
	Boats Owned	Indebtedness	Savings	Boats Purchased	Borrowings	Net Worth
	x_t	y_t	z_t	v_t	w_t	
	(number)	(dollars)	(dollars)	(number)	(dollars)	(dollars)
0	1	51,935	5,000	1	51,935	19,393
1	1	46,742	6,319	0	0	23,110
2	1	42,067	9,386	0	0	28,173
3	1	37,861	14,233	0	0	34,662
4	1	34,075	20,909	0	0	42,667
5	1	30,667	29,472	0	0	52,285
6	1	27,600	39,996	0	0	63,622
7	1	24,840	52,566	0	0	76,793
8	1	22,356	67,279	0	0	91,922
9	1	20,121	84,248	0	0	109,146
10	1	18,109	103,595	0	0	128,608

Table 5. Optimal Investment Strategy in Example 2

Year	States			Controls		Objective
	Boats Owned	Indebtedness	Savings	Boats Purchased	Borrowings	Net Worth
	x_t	y_t	z_t	v_t	w_t	
	(number)	(dollars)	(dollars)	(number)	(dollars)	(dollars)
0	1	75,000	5,000	1	75,000	25,785
1	1	67,500	3,308	0	0	27,556
2	1	60,750	3,691	0	0	30,822
3	1	54,675	6,160	0	0	35,663
4	1	49,208	10,745	0	0	42,168
5	1	44,287	17,490	0	0	50,436
6	1	39,858	26,454	0	0	60,573
7	2	121,146	0	1	85,274	67,517
8	2	109,031	17,379	0	0	89,060
9	3	188,372	0	1	90,244	109,681
10	3	169,535	34,172	0	0	150,148

Table 6. Conservative Investment Strategy in Example 2

Year	States			Controls		Objective
	Boats Owned	Indebtedness	Savings	Boats Purchased	Borrowings	Net Worth
	x_t	y_t	z_t	v_t	w_t	
	(number)	(dollars)	(dollars)	(number)	(dollars)	(dollars)
0	1	75,000	5,000	1	75,000	25,785
1	1	67,500	3,308	0	0	27,556
2	1	60,750	3,691	0	0	30,822
3	1	54,675	6,160	0	0	35,663
4	1	49,208	10,745	0	0	42,168
5	1	44,287	17,490	0	0	50,436
6	1	39,858	26,454	0	0	60,573
7	1	35,872	37,713	0	0	72,700
8	1	32,285	51,357	0	0	86,945
9	1	29,057	67,493	0	0	103,449
10	1	26,151	86,240	0	0	122,362

Table 7. Optimal Investment Strategy in Example 3

Year	States			Controls		Objective
	Boats Owned	Indebtedness	Savings	Boats Purchased	Borrowings	Net Worth
	x_t	y_t	z_t	v_t	w_t	
	(number)	(dollars)	(dollars)	(number)	(dollars)	(dollars)
0	1	93,750	5,000	1	93,750	30,982
1	1	84,375	5,188	0	0	35,499
2	1	75,937	8,167	0	0	42,082
3	1	68,344	13,965	0	0	50,844
4	1	61,509	22,631	0	0	61,909
5	1	55,358	34,234	0	0	75,416
6	2	150,214	0	1	100,392	85,223
7	2	135,193	24,503	0	0	114,825
8	3	224,010	0	1	102,336	143,649
9	4	316,196	0	1	114,588	192,218
10	4	284,576	70,204	0	0	272,609

Table 8. Conservative Investment Strategy in Example 3

Year	States			Controls		Objective
	Boats Owned	Indebtedness	Savings	Boats Purchased	Borrowings	Net Worth
	x_t	y_t	z_t	v_t	w_t	
	(number)	(dollars)	(dollars)	(number)	(dollars)	(dollars)
0	1	93,750	5,000	1	93,750	30,982
1	1	84,375	5,188	0	0	35,499
2	1	75,937	8,167	0	0	42,082
3	1	68,344	13,965	0	0	50,844
4	1	61,509	22,631	0	0	61,909
5	1	55,358	34,234	0	0	75,416
6	1	49,823	48,865	0	0	91,513
7	1	44,840	66,633	0	0	110,367
8	1	40,356	87,668	0	0	132,153
9	1	36,321	112,121	0	0	157,066
10	1	32,689	140,162	0	0	185,314

References

1. Russell G. Thompson, Richard W. Callen, and Lawrence C. Wolken. Optimal Investment and Financial Decisions for a Model Shrimp Fishing Firm. TAMU-SG-70-205, April 1970.

2. Russell G. Thompson, Richard W. Callen, and Lawrence C. Wolken. A Stochastic Investment Model for a Survival Conscious Fishing Firm. TAMU-SG-70-218, July 1970.

3. Robert R. Wilson, Russell G. Thompson, and Richard W. Callen. Optimal Investment and Financial Strategies in Shrimp Fishing. TAMU-SG-71-101, December 1970.

APPENDIX A

Glossary of Symbols

T	Length of planning period in years
z_t	Savings in year t
y_t	Indebtedness in year t
x_t	Number of boats owned in year t
v_t	Number of boats purchased in year t
w_t	Borrowings in year t
τ_t	Purchase price of a boat in year t
ψ_t	Technological depreciation factor: fractional value of a boat purchased in year t at the end of the planning period
κ	Maximum fraction of the boat's price that can be borrowed
β	Debt repayment rate
ζ	Interest rate on debt
ξ	Interest rate on savings
γ_t	Net price per pound of heads-off shrimp in year t
λ	Expected catch per boat in pounds of heads-off shrimp
θ_t	Operating cost per boat in year t
η_t	Sundry expense in year t
$g_t(v_i)$	Tax depreciation allowed in year t on boats purchased in year i
σ	Income tax rate